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Antisymmetric tensor spinor superfield representing a massless 2–3/2 supermultiplet

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Abstract. A 2-3/2 supermultiplet with the gravitino in the representation of an antisymmetric tensor spinor is discussed.

1. Introduction

Supersymmetry and S-matrix arguments call for $s=3/2$ as partner of the graviton, ruling out the $s=5/2$ alternative (Grisaru and Pendleton 1977, Grisaru *et al* 1977).

These results have also been confirmed algebraically for the $s=5/2$ symmetric formulation (Aragone and Deser 1979, Berends *et al* 1980) as well as for the $s=5/2$ vierbein representation (Aragone and Deser 1980a,b) showing agreement with predictions made in a different approach that uses methods of topology (Christensen and Duff 1979).

With the $s=3/2$ field in the $(1, 1/2)$ Lorentz representation of Rarita and Schwinger this leads to supergravity in its well known form (Deser and Zumino 1976, Freedman *et al* 1976). Here we discuss the effect caused by changing the representation of the gravitino, that is we consider a 2-3/2 supermultiplet where the fermion comes in the $(0, 3/2)$ representation of the Lorentz group.

The investigations are done for the linearised version of the theory. We find that the action derived from the basic superfield has similarities with that one of linearised conformal supergravity (Ferrara and Zumino 1978). In particular the analysis of the physical degrees of freedom of the theory shows that ghosts are present. But unlike in the superconformal case where the ghosts are caused by higher derivatives, here the unphysical particles decouple in a supercovariant way. The Fock space splits into two disjoint pieces on which the algebra is realised separately.

2. Construction of the multiplet and action functional

We start off with the construction of the basis superfield. The spin-3/2 representation in question is contained in the γ-transverse projection $\psi_{\hat{\mu}\hat{\nu}}$ of an antisymmetric

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second-rank tensor-(Majorana)-spinor $\psi_{\mu\nu}$ which we decompose as follows†:

$$\psi_{\mu\nu} = \psi_{\tilde\mu\tilde\nu} - \gamma_{\mu\nu\sigma}\psi^{\sigma} = \psi_{\tilde\mu\tilde\nu} + \gamma_{[\mu}\psi_{\nu]} - 2\sigma_{\mu\nu}\gamma^{\sigma}\psi_{\sigma}. \tag{1}$$

Here $\psi_{\nu} = \frac{1}{6}(\gamma_{\nu\rho\sigma} + \eta_{\nu\sigma}\gamma_{\rho})\psi_{\rho\sigma}$ is the Rarita–Schwinger field.

The superfield that carries the spinor $\psi_{\mu\nu}$ as a component field is obtained from the $s = 1/2$ chiral superfield W_A (as found by Grimm *et al* (1978) and Wess (1978) for the vector multiplet) by adding the corresponding index structure:

$$W \rightarrow W_{\mu\nu}.$$

According to Fischler (1979) this superfield has no local extension, but its $(0, 3/2)$ projection does which we denote by

$$W_{\tilde\mu\tilde\nu} = \{W_{\mu\nu}\}_{\tilde\mu\tilde\nu}.$$

This projection we write out explicitly (in the Weyl representation of the spinors) as it carries the fundamental multiplet:

$$W_{\tilde\mu\tilde\nu} = \{\psi_{\mu\nu} + \mathrm{i}A_{\mu\nu}\theta + S_{\alpha\beta,\mu\nu}\sigma^{\alpha\beta}\theta + 2\theta^{2}\partial_{[\mu}\psi_{\nu]}\}_{\tilde\mu\tilde\nu}. \tag{2}$$

We learn that besides the $(0, 3/2)$ representation the Rarita–Schwinger field is also present. The graviton is in the $(3/2, 1/2)$ representation of the vierbein connection represented by the curvature tensor

$$S_{\alpha\beta,\mu\nu} = \partial_{\alpha}\omega_{\beta,\mu\nu} - \partial_{\beta}\omega_{\alpha,\mu\nu}.$$

The antisymmetric pseudotensor $A_{\mu\nu}$ is an auxiliary field. As in the case of the vectormultiplet $W_{\tilde\mu\tilde\nu}$ is the field strength of a real superfield potential $V_{\mu\nu}$ with Abelian gauge invariances

$$\delta\psi_{\tilde\mu\tilde\nu} = 0 \qquad \delta\psi_{\mu} = \partial_{\mu}\varepsilon \qquad \delta A_{\mu\nu} = 0 \qquad \delta\omega_{\mu,\alpha\beta} = 0 \tag{3a}$$

and

$$\delta\psi_{\tilde\mu\tilde\nu} = 0 \qquad \delta\psi_{\mu} = 0 \qquad \delta A_{\mu\nu} = 0 \qquad \delta\omega_{\mu,\alpha\beta} = \partial_{\mu}\varepsilon_{\alpha\beta} \tag{3b}$$

where ε is a Majorana spinor and $\varepsilon_{\alpha\beta}$ an antisymmetric tensor. The superalgebra is realised on the fields according to the supertransformations with parameter α (see also Curtright 1979)

$$\delta\omega_{\alpha,\mu\nu} = \mathrm{i}\bar\alpha\gamma_{\alpha}\psi_{\tilde\mu\tilde\nu} - \mathrm{i}\bar\alpha\gamma_{\alpha}\gamma_{\mu\nu\sigma}\psi^{\sigma} \tag{4a}$$

$$\delta A_{\mu\nu} = \bar\alpha\gamma_{5}\partial\!\!\!/(\psi_{\tilde\mu\tilde\nu} - \gamma_{\mu\nu\sigma}\psi^{\sigma}) \tag{4b}$$

$$\delta\psi_{\tilde\mu\tilde\nu} = \{-\sigma^{\alpha\beta}\alpha S_{\alpha\beta,\mu\nu} + \gamma_{5}\alpha A_{\mu\nu}\}_{\tilde\mu\tilde\nu}. \tag{4c}$$

$$\begin{aligned}\delta\psi_{\nu} = &-\tfrac{1}{3}\gamma_{5}\gamma^{\mu}\alpha S^{*}_{\mu,\nu} + \tfrac{1}{6}\gamma_{\nu}\alpha S - \tfrac{1}{3}\gamma^{\mu}\alpha S_{\nu,\mu} - \tfrac{1}{6}\gamma^{\mu}\alpha S_{\mu,\nu}\\ &+\tfrac{1}{6}\gamma_{5}\gamma^{\mu}\alpha {}^{*}S_{\mu,\nu} + \tfrac{1}{3}\gamma^{\mu}\alpha A^{*}_{\mu\nu} - \tfrac{1}{6}\gamma_{5}\gamma^{\mu}\alpha A_{\mu\nu}\end{aligned} \tag{4d}$$

where we have introduced the definitions

$$S_{\alpha,\beta} = S^{\mu}{}_{\alpha,\mu\beta}, \qquad S = S^{\alpha}{}_{,\alpha}, \qquad {}^{*}S_{\mu\nu,\sigma\tau} = \tfrac{1}{2}\varepsilon_{\mu\nu}^{\ \ \alpha\beta}S_{\alpha\beta\sigma\tau}, \qquad {}^{*}S_{\nu,\sigma} = {}^{*}S^{\mu}{}_{\nu,\mu\sigma}.$$

The action for this multiplet invariant under (3) and (2) is given by the last component of W^2

$$I = \tfrac{1}{4}\int \mathrm{d}^{4}x\, \mathrm{d}^{2}\theta W^{\tilde\mu\tilde\nu}W_{\tilde\mu\tilde\nu} + \mathrm{HC} = I_{3/2} + I_{2} \tag{5a}$$

† Our γ matrices are real, $\{\gamma_{\mu}, \gamma_{\nu}\} = 2\eta_{\mu\nu}$, $\eta_{\mu\nu} = (-, +++)$, $\gamma_{5} = \gamma_{0}\gamma_{1}\gamma_{2}\gamma_{3}$, $\varepsilon^{0123} = +1$, $\gamma^{\mu\nu\rho} = \varepsilon^{\mu\nu\rho\sigma}\gamma_{\sigma}\gamma_{5}$. Tilded indices indicate γ-transverse projection, and the symbol $[\mu, \nu] \equiv \mu\nu - \nu\mu$.

where

$$I_{3/2} = 2\mathrm{i}\int \mathrm{d}^4x\, \bar{\psi}^{\tilde{\mu}\tilde{\nu}}\partial_\mu\psi_\nu \tag{5b}$$

and

$$I_2 = \tfrac{1}{12}\int \mathrm{d}^4x(S^{\mu\nu,\alpha\beta}S_{\mu\nu,\alpha\beta} + 2S^{\mu\nu,\alpha\beta}S_{\mu\alpha,\nu\beta} - 2S^{\mu,\nu}S_{\mu,\nu} - 2{}^*\dot{S}^{[\mu,\nu]}A_{\mu\nu} - 2A^{\mu\nu}A_{\mu\nu}). \tag{5c}$$

This action constitutes the basis for our investigations. The following analysis we carry out separately for the fermionic and bosonic part.

3. The fermion action

First we should like to comment on the special structure of $I_{3/2}$. We find that this is the only way of writing a kinetic term for the $(0, 3/2)$ representation (except for additional Rarita–Schwinger terms, which we discuss later) as it is easily seen in the Weyl representation. There the $(0, 3/2)$ field is given by a totally symmetric spinor ψ_{ABC}. The derivative ∂_μ becomes $\partial_{A\dot{A}}$, hence the open indices in the expression $\psi_{ABC}\partial^{A\dot{A}}$ (which must be part of the kinetic term in the Lagrangian) can only be closed up by a spinor with index structure $\chi_{\dot{A}}^{BC}$ which is the Rarita–Schwinger field. Similarly one finds that the $(0, 3/2)$ field cannot possess a gauge mechanism associated with an $s = 1/2$ parameter, for a gauge transformation

$$\delta\psi_{ABC} = (\partial_{A\dot{B}}\bar{\xi}^{\dot{B}})_{(ABC)}$$

would require an object with two undotted symmetric indices which does not exist (the only quantity with two undotted indices is ε_{AB}, but it is antisymmetric)†. This shows that $I_{3/2}$ as it stands exhausts already all possibilities.

In order to determine the physical degrees of freedom we consider the functional

$$Z[j_{\mu\nu}, j_\nu] = \int \mathrm{D}\psi_{\mu\nu}\mathrm{D}\psi_\nu\delta(G^{\mathrm{C}})\exp\Big(\mathrm{i}\int \mathrm{d}^4x\ \mathrm{i}\{2\bar{\psi}^{\tilde{\mu}\tilde{\nu}}\partial_\mu\psi_\nu + \bar{\psi}^{\tilde{\mu}\tilde{\nu}}j_{\tilde{\mu}\tilde{\nu}} + 4\bar{\psi}^\mu j_{\tilde{\mu}}\}\Big). \tag{6}$$

The external sources $j_{\tilde{\mu}\tilde{\nu}}$ and $j_{\tilde{\mu}}$ are γ-traceless and in addition one has

$$\partial^\mu j_{\tilde{\mu}} = 0$$

due to gauge invariance. We pick the Coulomb gauge $G^{\mathrm{C}} = \partial_i\psi_i = 0$. The antisymmetric current $j_{\tilde{\mu}\tilde{\nu}}$ (as well as the field $\psi_{\tilde{\mu}\tilde{\nu}}$) can be represented by three-dimensional γ-transverse spatial vector spinors $j_{\tilde{i}\tilde{0}}$ $(\psi_{\tilde{i}\tilde{0}})$

$$\begin{aligned}(j, \psi)_{\tilde{i}\tilde{j}} &= \gamma_0\gamma_i(j, \psi)_{\tilde{j}\tilde{0}} - \gamma_0\gamma_j(j, \psi)_{\tilde{i}\tilde{0}}\\ \gamma_i(j, \psi)_{\tilde{i}\tilde{0}} &= 0.\end{aligned} \tag{7}$$

A further decomposition of the spatial vector spinors $\gamma_0 j_i$, $\gamma_0\psi_i$, $\psi_{\tilde{i}\tilde{0}}$ and $j_{\tilde{i}\tilde{0}}$ as described by Aragone and Deser (1980a,b) is useful:

$$\begin{aligned}\gamma_0(\psi, j)_i &= (\psi, j)_i^{\mathrm{TT}} + \tfrac{1}{2}\gamma_i^{\mathrm{T}}(\psi, j)^{\mathrm{T}l} + \rho_i(\psi, j)^{\mathrm{L}}\\ (\psi, j)_{\tilde{i}\tilde{0}} &= (\psi, j)_{i0}^{\mathrm{TT}} + \tfrac{1}{2}\gamma_i^{\mathrm{T}}(\psi, j)_0^{\mathrm{T}l} + \rho_i(\psi, j)_0^{\mathrm{L}}\end{aligned} \tag{8}$$

† Moreover, irreducibility of a totally symmetric $\delta\psi_{ABC}$ makes impossible its algebraic connection with either $\partial_{A\dot{B}}\xi_{\dot{C}}$ or $\partial_{A\dot{B}}\xi_C$.

where $\rho_i=(-\Delta)^{-1/2}\partial_i$, $\gamma_i^{\mathrm{T}}=(\delta_{ij}+\rho_i\rho_j)\gamma_j=\mathbb{P}_{ij}\gamma_j$ and the gauge invariant variables $(\psi, j)^{\mathcal{T}\mathrm{T}}$ are transverse and three-dimensional γ-transverse:

$$\gamma_i\psi_i^{\mathcal{T}\mathrm{T}}=0=\rho_i\psi_i^{\mathcal{T}\mathrm{T}}.$$

Insertion of (8) in (6) and integration over Lagrange multipliers yields

$$Z[j_{\bar{\mu}\bar{\nu}}, j_\nu]=\int \mathrm{D}\psi_i^{\mathcal{T}\mathrm{T}}\mathrm{D}\psi_{i0}^{\mathcal{T}\mathcal{T}}\exp\Big(\mathrm{i}\int \mathrm{d}^4x\,\mathrm{i}\{2\bar{\psi}_i^{\mathcal{T}\mathcal{T}}\not{\partial}\psi_{i0}^{\mathcal{T}\mathrm{T}}$$
$$+4\bar{\psi}_i^{\mathcal{T}\mathrm{T}}j_i^{\mathcal{T}\mathrm{T}}-4\bar{\psi}_{i0}^{\mathcal{T}\mathrm{T}}j_{i0}^{\mathcal{T}\mathrm{T}}+12(\bar{j}^{\mathrm{T}\prime}+\bar{j}^{\mathrm{L}}\not{\rho})(-\Delta)^{-1/2}j_0^{\mathrm{L}}\}\Big). \tag{9}$$

From (9) we learn that two $s=3/2$ variables remain. All lower spins do not contribute. The way, however, the 3/2 fields appear here indicates the presence of a ghost. To reveal it one simply may redefine the variables according to

$$\psi_i^{\mathcal{T}\mathrm{T}}=2^{-1/2}(\lambda+\varphi)_i^{\mathcal{T}\mathrm{T}} \qquad \text{and} \qquad \psi_{i0}^{\mathcal{T}\mathrm{T}}=2^{-1/2}(\lambda-\varphi)_i^{\mathcal{T}\mathrm{T}}. \tag{10}$$

$\varphi_i^{\mathcal{T}\mathrm{T}}$ is a ghost field.

Another way of saying this is that one source appears as antisource of the other that follows upon performing the remaining integrations:

$$Z[j_{\bar{\mu}\bar{\nu}}, j_\nu]=\exp\Big(\mathrm{i}\int \mathrm{d}^4x\,\mathrm{i}\{8\bar{j}_{i0}^{\mathcal{T}\mathrm{T}}\not{\partial}^{-1}j_i^{\mathcal{T}\mathrm{T}}+12(\bar{j}^{\mathrm{T}\prime}+\bar{j}^{\mathrm{L}}\not{\rho})(-\Delta)^{-1/2}j_0^{\mathrm{L}}\}\Big). \tag{11}$$

This shortcoming is a feature of the $(0, 3/2)$ representation which requires an off-diagonal structure in the Lagrangian for its dynamics.

Leaving aside supersymmetry one may try to improve the situation by adding further pieces to $I_{3/2}$. It turns out that diagonal Rarita–Schwinger terms only represent new contributions which, however, worsen the theory, for they generate further $s=1/2$ excitations and leave the ghosts (see also Townsend 1980, Deser *et al* 1981)

4. The boson action

I_2 contains the graviton in the (3/2, 1/2) representation of the vierbein connection. From the results of Sezgin and van Niewenhuizen (1980) we know that there is no possibility of having a propagating massless spin-2 field in this representation which is ghost-free. Therefore we are not surprised to find it here as a supersymmetry companion of the fermionic ghost action $I_{3/2}$.

In order to identify the physical fields we proceed as above. The generating functional reads

$$Z[j_{\mu,\alpha\beta}]=\int \mathrm{D}\omega_{\mu,\alpha\beta}\delta(G^{\mathrm{C}})\exp\Big(\mathrm{i}\int \mathrm{d}^4x\{\tfrac{1}{12}S^{\mu\nu,\alpha\beta}S_{\mu\nu,\alpha\beta}$$
$$-\tfrac{1}{4}S^{\mu,\nu}S_{\mu,\nu}+\tfrac{1}{6}S^{\mu\nu,\alpha\beta}S_{\mu\alpha,\nu\beta}+\tfrac{1}{12}S^{\mu,\nu}S_{\mu,\nu}+\tfrac{1}{4}\omega_{\mu,\alpha\beta}j^{\mu,\alpha\beta}\}\Big) \tag{12}$$

where $j_{\mu,\alpha\beta}=-j_{\mu,\beta\alpha}$ and $\partial^\mu j_{\mu,\alpha\beta}=0$.

For the evaluation of $Z[j_{\mu,\alpha\beta}]$ we introduce the 3+1 induced variables

$$\omega_{0,0i}=l_i \qquad \omega_{0,ij}=\varepsilon_{ijk}K_k \qquad \omega_{i,0j}=\varepsilon_{ijk}a_k+f_{ij} \qquad \omega_{i,jk}=-\delta_{i[j}b_{k]}+\varepsilon_{jkl}h_{li}.$$

f_{ii} and h_{ii} are symmetric three-dimensional tensors both carrying massless spin 2. The Coulomb gauge is

$$G^{C} = \partial_i f_{ij} = 0 = \partial_i h_{ij}.$$

Further decomposition of the vectors and tensors into their standard transverse and longitudinal variables leads in particular for the tensors (in the chosen gauge) to

$$(h, f)_{ij} = (h, f)^{TT}_{ij} + \tfrac{1}{2}\mathbb{P}_{ij}(h, f)_{kk}. \tag{13}$$

$(h, f)^{TT}_{ij}$ are gauge invariant variables.

Correspondingly the sources can be described by

$$j_{0,0i} = j_i$$

$$j_{0,ij} = \varepsilon_{ijk} j^{*}_{k} \qquad j_{i,0j} = \varepsilon_{ijk}\hat{j}_k + j^{1}_{ij} \qquad j_{i,jk} = -\delta_{i[j}\check{j}_{k]} + \varepsilon_{jkl} j^{2}_{il}, \qquad (j^{1,2}_{ij} = j^{1,2}_{ji}).$$

As the 24 bosonic degrees of freedom contain 6 multipliers associated with 6 constraints, there will be no more than $12 = 24 - 2\times 6$ propagating degrees of freedom. In fact we find after integration over the Lagrange multipliers

$$\begin{aligned} Z[j_{\mu,\alpha\beta}] = \int \mathrm{D}f^{TT}_{ij} Dh^{TT}_{ij} \exp\Big(\mathrm{i}\int \mathrm{d}^4x\{&\tfrac{1}{2}f^{TT}_{ij}\Box f^{TT}_{ij} - \tfrac{1}{2}h^{TT}_{ij}\Box h^{TT}_{ij} + \tfrac{1}{8}(j^{T} - 2\check{j}^{T})\Delta^{-1}(j^{T} - 2\check{j}^{T}) \\ &- \tfrac{1}{8}(j^{*T} + 2\hat{j}^{T})\Delta^{-1}(j^{*T} + 2\hat{j}^{T}) + \tfrac{3}{64}(j^{L} - 2\check{j}^{L})\Delta^{-1}(j^{L} - 2\check{j}^{L}) \\ &- \tfrac{3}{64}(j^{*L} + 2\hat{j}^{L})\Delta^{-1}(j^{*L} + 2\hat{j}^{L}) + \tfrac{1}{4} j^{2TT}_{ij} h^{TT}_{ij} - j^{1TT}_{ij} f^{TT}_{ij}\}\Big). \end{aligned} \tag{14}$$

These are two propagating spin-2 fields, one of which is a ghost. All lower spins do not propagate. This is also what one would expect from supersymmetry and it confirms the result of Sezgin and van Niewenhuizen (1980).

In order to show complete analogy with the fermionic action we introduce variables $f^{\pm}_{ij} = (f \pm h)_{ij}$ and redefine the sources by $j^{\pm}_{ij} = (j^{2} \pm j^{1})_{ij}$. The last two integrations in (14) then have the result

$$Z[j_{\mu,\alpha\beta}] = \exp\Big(\mathrm{i}\int \mathrm{d}^4x\Big\{\tfrac{1}{32} j^{+TT}_{ij} \tfrac{1}{\Box} j^{-TT}_{ij} + \sum_a c_a j_a \Delta^{-1} j_a\Big\}\Big). \tag{15}$$

This is the bosonic counterpart to (10).

5. Ghost decoupling

As the ghosts found in the analysis above have a simple structure we are able to separate them from the physical particles in a supercovariant way. To see this we write the fields in terms of their Fourier expansions in a special frame of reference:

$$(f^{\overset{+}{(-)}}_{(x)})^{TT}_{ij} = \int \frac{\mathrm{d}^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_p}} \sum_{s=\pm} (\varepsilon_{ij}(s, \vec{p}) a_{\overset{1}{(2)}}(s, \vec{p})\, \mathrm{e}^{\mathrm{i}px} + \mathrm{HC})$$

$$\begin{aligned} (\psi^{\overset{+}{(-)}})^{TT}_{i} &\equiv (1 \overset{-}{\scriptstyle(+)} \gamma_5)\psi^{TT}_{i0} - (1 \overset{+}{\scriptstyle(-)} \gamma_5)\gamma_0 \psi^{TT}_{i} \\ &= \int \frac{\mathrm{d}^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_p}} \sum_{s=\pm} (\varepsilon_i(s, \vec{p})\chi(s, \vec{p}) b_{\overset{1}{(2)}}(s, \vec{p})\, \mathrm{e}^{\mathrm{i}px} + \mathrm{HC}). \end{aligned} \tag{16}$$

Here

$$\varepsilon_{ij}(\pm, \vec{p}) = \varepsilon_i(\pm, \vec{p})\varepsilon_j(\pm, \vec{p})$$

where $\varepsilon_i(\pm)$ is a complex polarisation vector associated with the helicity operator $b(\pm)$ and it obeys the identities

$$\sum_{s=\pm} \varepsilon_i(s)\varepsilon_j^*(s) = \mathbb{P}_{ij}, \qquad \varepsilon_i(s)\varepsilon_i^*(s') = \delta_{ss'}, \qquad p_i\varepsilon_i(\pm, \vec{p}) = 0. \tag{17}$$

In a general frame one has (de Witt 1964)

$$\varepsilon^\mu(s)\varepsilon_\mu^*(s') = \delta_{ss'}, \qquad p^\mu\varepsilon_\mu(s) = 0 = \bar{p}^\mu\varepsilon_\mu(s)$$

$$\sum_{s=\pm} \varepsilon^\mu(s)\varepsilon^{\mu *}(s) = \eta^{\mu\nu} - (p\bar{p})^{-1}(p^\mu\bar{p}^\nu + p^\nu\bar{p}^\mu)$$

$$\bar{p}_\mu = p_\mu + 2\eta_\mu\eta^\nu p_\nu, \qquad \eta^\mu\eta_\mu = -1.$$

The only non-vanishing canonical brackets of the helicity operators are

$$\begin{aligned}&[a_1(s, \vec{p}), a_2^+(s', \vec{p}')] = \delta_{ss'}\delta^3(\vec{p} - \vec{p}')\\ &\{b_1(s, \vec{p}), b_2^+(s', \vec{p}')\} = \delta_{ss'}\delta^3(\vec{p} - \vec{p}').\end{aligned} \tag{18}$$

The four-momentum becomes

$$\begin{aligned}P_\mu = \int (\mathrm{d}^3p)p_\mu \sum_s (&a_1^+(s, \vec{p})a_2(s, \vec{p}) + a_2^+(s, \vec{p})a_1(s, \vec{p})\\ &+ b_1^+(s, \vec{p})b_2(s, \vec{p}) + b_2^+(s, \vec{p})b_1(s, \vec{p}))\end{aligned} \tag{19}$$

and the supersymmetry generators are represented by

$$Q = -\mathrm{i}\int \mathrm{d}^3p \sum_{s=\pm} [(a_1^+(s, \vec{p})b_2(s, \vec{p}) + a_2^+(s, \vec{p})b_1(s, \vec{p}))\varepsilon^{*\mu}(s, \vec{p})\gamma_\mu\chi(s, \vec{p}) - \mathrm{HC}] \tag{20}$$

with

$$\{Q, Q\} = 2\not{p}\gamma^0, \qquad p^0 = E > 0.$$

Therefore one can immediately diagonalise the Hamiltonian by the linear transformation

$$A = \frac{1}{\sqrt{2}}(a_1 + a_2) \qquad \alpha = \frac{1}{\sqrt{2}}(a_1 - a_2)$$

$$B = \frac{1}{\sqrt{2}}(b_1 + b_2) \qquad \beta = \frac{1}{\sqrt{2}}(b_1 - b_2)$$

when it splits into two pieces

$$H_{\text{particle}} = \int (\mathrm{d}^3p)E_p \sum_s (A^+(s, \vec{p})A(s, \vec{p}) + B^+(s, \vec{p})B(s, \vec{p}))$$

and

$$H_{\text{ghost}} = \int (\mathrm{d}^3p)(-E_p) \sum_s (\alpha^+(s, \vec{p})\alpha(s, \vec{p}) + \beta^+(s, \vec{p})\beta(s, \vec{p})). \tag{21}$$

Since the supertransformations do not mix ghosts with particles, it follows from the commutator

$$[A(s, \vec{p}), Q] = -i\varepsilon^{\mu *}(s, \vec{p})\gamma_\mu \chi(s, \vec{p})B(s, \vec{p}) \tag{22}$$

(same for α and β) that the total Fock space decomposes into two invariant subspaces of which one represents the ghosts with negative-norm kets. This splitting is due to the simple attachment of the ghosts to the particles and does not occur in the more sophisticated conformal case (Ferrara and Zumino 1978).

6. Conclusion

We found that a change in the representation of the gravitino to $(0, 3/2)$ leads to a theory totally different from supergravity. In particular as the fermion representation comes with an $s = 3/2$ ghost field which is necessary for it to propagate, supersymmetry calls for a ghost action for the graviton as well. The latter is therefore quadratic in the curvature and belongs to a class of actions investigated also by Sezgin and van Niewenhuizen (1980).

From our results we learn also that in conformal supergravity where the Rarita–Schwinger field strength enters through its $(0, 3/2)$ projection (Ferrara and Zumino 1978) the ghost structure originates from a superposition of higher-derivative ghosts and those arising from the fermion representation. Finally gauging this antisymmetric tensor spinor theory would mean that the gauge transformation $(3a)$ combines with the transformation $(4d)$ to

$$\delta\psi_\mu = \kappa^{-1}\mathrm{D}_\mu\alpha + (4d) + \dots. \tag{23}$$

A gauge variation of the local action $I_{3/2}$ then generates the term

$$\delta I_{3/2} \sim \int (\mathrm{d}^4x) \frac{1}{\kappa} \bar{\psi}^{\hat{\mu}\hat{\nu}} \sigma^{ab} \alpha R_{\mu\nu,ab} \tag{24}$$

which has been encountered already in all other antisymmetric tensor theories (Deser and Witten 1980, Deser *et al* 1981, Townsend 1980) where it gave rise to inconsistencies. Although here the Riemann tensor is part of the full local action it is not clear yet whether this theory can escape those difficulties.

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